

March 6, 2013

The Born series for S -wave quartet nd scattering at small cutoff values

Shung-Ichi Ando¹

*Department of Physics Education, Daegu University, Gyeongsan 712-714,
Republic of Korea*

Perturbative expansions, the Born series, of the scattering length and the amplitude of S -wave neutron-deuteron scattering for spin quartet channel below deuteron breakup threshold are studied in pionless effective field theory at small cutoff values. A three-body contact interaction is introduced when the integral equation is solved using the small cutoffs. After renormalizing the three-body interaction by using the scattering length, we expand the integral equation, as the Born series, around weak coupling limit and unitary limit. We find that the scattering length and the phase shift are considerably well reproduced with a few terms of the Born series expanded around the unitary limit at the small cutoff values.

PACS numbers: 03.65.Kk, 25.10.+s, 25.40.Dn.

¹<mailto:sando@daegu.ac.kr>

1. Introduction

After Weinberg suggested the application of chiral perturbation theory, a low energy effective field theory of QCD, to nuclear force [1], a lot of works have been done during last two decades. For reviews, see, e.g., Refs. [2, 3, 4, 5]. Meanwhile, new facilities for the rare isotope beams, in which exotic nuclei near the neutron (proton) drip line can be created, provide us a new opportunity for investigation [6, 7, 8]. Such a limit close to the drip line where the single neutron (proton) separation energy vanishes can be theoretically recognized as unitary limit (to be discussed below), and thus the unitary limit would be a good theoretical starting point to study the exotic nuclei near the nuclear drip line. In this work, we discuss a perturbative expansion, as the Born series, of an integral equation around the unitary limit (as well as weak coupling limit) by sending the cutoff parameter to small values. We study it in S -wave neutron-deuteron (nd) scattering for spin quartet channel in pionless effective field theory (EFT).

Pionless EFT [9, 10] is one of low energy EFTs for few nucleon systems, in which one chooses a typical momentum scale Q smaller than the pion mass, m_π , and thus the pions are regarded as heavy degrees of freedom and integrated out of effective Lagrangian. Thus the large scale of the pionless EFT is $\Lambda \simeq m_\pi$, and the theory provides us a systematic expansion scheme in terms of Q/Λ . In applications of the pionless EFT, much more attention has been paid to S -wave nd scattering for spin doublet ($S = 1/2$) channel because the one-nucleon-exchange interaction becomes “singular” [11, 12]. To control the singularity one promotes a three-body contact interaction to leading order (LO), and the coupling constant of the contact interaction exhibits so called limit-cycle when the scale parameter Λ is sent to the asymptotic limit. This behavior is understood to be associated with “Efimov effect”. On the other hand, an application of pionless EFT to S -wave nd scattering for spin quartet ($S = 3/2$) channel is regarded well known because the phase shift δ_0 of the quartet nd scattering is almost perfectly described by two effective range parameters in two-body deuteron channel, and no cutoff dependence is reported [13, 14].

In this work, we revisit the S -wave quartet nd scattering below deuteron breakup threshold in pionless EFT and reduce the cutoff parameter Λ smaller than m_π . The small scale limit might be interesting for studying, e.g., a relation between the pionless EFT and a Halo/Cluster EFT [15] in which one may choose a typical scale smaller than the deuteron breakup momentum, and the deuteron is never broken into two nucleons and could be regarded as a cluster or an elementary field [16].

When the value of Λ is sent to smaller than m_π , the cutoff dependence emerges even in the quartet channel. Thus we introduce a three-body contact interaction when we solve the integral equation using the small cutoff values. After renormalizing the strength of the three-body interaction by using the scattering length a_4 of the S -wave quartet nd scattering we expand the integral equation for the scattering length a_4 and the scattering amplitude in terms of the Born series up to next-to-next-to leading order (NNLO)². We expand the Born series around so called trivial and non-trivial fixed point studied in the renormalization group analysis by Birse, McGovern, and Richardson [17]. The trivial

²We note that this is not an expansion scheme in EFT, but in terms of the Born series. We do not have a clear expansion parameter as Q/Λ .

fixed point corresponds to weak coupling limit where all interactions vanish, whereas the non-trivial fixed point does to unitary limit where the scattering length becomes infinity or the binding energy vanishes. Around the unitary limit, the Born series is expanded around the inverse of the amplitude [18]. We find that, by reducing the cutoff value, both a_4 and δ_0 are considerably well reproduced by a few terms of the Born series expanded around the unitary limit.

This work is organized as the following. In Sec. 2, effective Lagrangian is displayed. In Sec. 3, integral equations for the scattering length and the amplitude are given, and the coupling of the three-body interaction is renormalized by the scattering length a_4 . In Sec. 4, the scattering length and the amplitude are expanded in terms of the Born series around the weak coupling limit and the unitary limit, and the numerical results are obtained. Finally, in Sec. 5, the discussion and conclusions are presented.

2. Effective Lagrangian

The effective Lagrangian for the S -wave quartet nd scattering in pionless EFT reads [10, 12]

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_t + \mathcal{L}_3, \quad (1)$$

where \mathcal{L}_N is the standard one-nucleon Lagrangian in the heavy-baryon formalism,

$$\mathcal{L}_N = N^\dagger \left\{ iv \cdot D + \frac{1}{2m_N} [(v \cdot D)^2 - D^2] + \dots \right\} N, \quad (2)$$

where v^μ is a velocity vector satisfying a condition $v^2 = 1$, D_μ is the covariant derivative, and m_N is the nucleon mass. \mathcal{L}_t is dibaryon effective Lagrangian for 3S_1 state,

$$\mathcal{L}_t = \sigma_t t_i^\dagger \left\{ iv \cdot D + \frac{1}{4m_N} [(v \cdot D)^2 - D^2] + \Delta_t \right\} t_i - y_t \left\{ t_i^\dagger [N^T P_i^{({}^3S_1)} N] + h.c. \right\}, \quad (3)$$

where σ_t is a sign factor, $\sigma_t = \pm 1$. t_i is a dibaryon field for spin triplet (3S_1) state. D_μ is the covariant derivative for the dibaryon field. Δ_t is the mass difference between the dibaryon and two nucleons, $m_t = 2m_N + \Delta_t$. y_t is a coupling constant for dibaryon-nucleon-nucleon (dNN) vertex. $P_i^{({}^3S_1)}$ is a projection operator for two nucleons in the 3S_1 state,

$$P_i^{({}^3S_1)} = \frac{1}{\sqrt{8}} \tau_2 \sigma_2 \sigma_i, \quad (4)$$

where τ_a and σ_i are Pauli matrices for isospin and spin, respectively. Those two constants, Δ_t and y_t , and the sign factor $\sigma_t (= -1)$ are determined by two effective range parameters, deuteron binding momentum γ and effective range ρ_d , in NN scattering for 3S_1 channel.

We obtain \mathcal{L}_3 , an effective Lagrangian of three-body contact interaction for the S -wave spin quartet nd state, as

$$\mathcal{L}_3 = -\frac{m_N}{6} g(\Lambda) y_t^2 N^\dagger [(\vec{\sigma} \cdot \vec{t})^\dagger (\vec{\sigma} \cdot \vec{t}) + 3(\vec{\sigma} \cdot \vec{t})(\vec{\sigma} \cdot \vec{t})^\dagger] N. \quad (5)$$

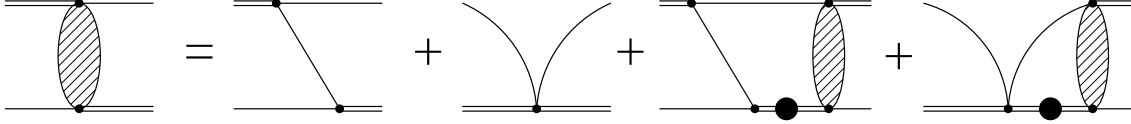


Figure 1: Diagrams for S -wave nd scattering in spin quartet ($S = 3/2$) channel. A shaded blob denotes scattering amplitude a line (a double-line) nucleon (dibaryon), and a double line with a filled circle dressed dibaryon propagator (see Fig. 2).

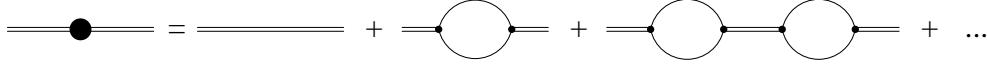


Figure 2: Diagrams for a dressed dibaryon propagator. See the caption of Fig. 1 as well.

We note that the three-body interaction is a higher order term in the quartet nd scattering in pionless EFT, and it is not necessary to be introduced when the value of the cutoff Λ is about m_π or larger, $\Lambda \geq m_\pi$, at leading order (LO). We introduce it, however, so as to reproduce the scattering length a_4 in the quartet channel (and the strength of the coupling $g(\Lambda)$ is adjusted) when the Λ value is reduced smaller than m_π , $\Lambda < m_\pi$.

3. Integral equations

3.1 Integral equation of S -wave nd scattering amplitude for quartet channel

Diagrams for S -wave nd scattering for the spin quartet ($S = 3/2$) state are given in Fig. 1, whereas those for the two-body part in Fig. 1, the dressed dibaryon propagator, are given in Fig. 2. The integral equation of the S -wave quartet nd scattering without the three-body contact interaction is known as [14]

$$t(p, k) = -\frac{8\pi}{m\rho_d} K^{(a)}(p, k; E) - \frac{2}{\pi} \int_0^\Lambda dq K^{(a)}(p, q; E) \frac{q^2 t(q, k)}{-\gamma + \frac{1}{2}\rho_d \left(\gamma^2 - \frac{3}{4}q^2 + mE \right) + \sqrt{\frac{3}{4}q^2 - mE}}, \quad (6)$$

where $t(p, k)$ is the half-off-shell scattering amplitude and p (k) is the magnitude of off-shell final state (on-shell initial state) relative three momentum in CM frame. $K^{(a)}(p, q; E)$ is the one-nucleon-exchange interaction,

$$K^{(a)}(p, q; E) = \frac{1}{2pq} \ln \left(\frac{p^2 + q^2 + pq - mE}{p^2 + q^2 - pq - mE} \right), \quad (7)$$

where $E = -\frac{\gamma^2}{m_N} + \frac{3}{4m_N}k^2$: γ and ρ_d are the deuteron binding momentum and the effective range, respectively. A sharp cutoff Λ is introduced in the loop integral.

We note that because a singularity at $q \simeq 197 \text{ MeV}$ which represents an unphysical deeply bound state appears in the propagator, the dressed dibaryon propagator is usually expanded in terms of the effective range, ρ_d , to avoid an effect from the unphysical bound state. However, we are interested in reducing the cutoff value, less than $\Lambda = 197 \text{ MeV}$, so we do not expand the dressed dibaryon propagator in the integral equation in Eq. (6).

Thus the on-shell T-matrix is given by

$$T(k, k) = \sqrt{Z_d} t(k, k) \sqrt{Z_d}, \quad (8)$$

where Z_d is the deuteron wavefunction normalization factor, $Z_d = \gamma \rho_d / (1 - \gamma \rho_d)$. In terms of the half-off-shell scattering amplitude $T(p, k)$, we have the integral equation as

$$\begin{aligned} T(p, k) = & -\frac{8\pi}{m_N \rho_d} Z_d K^{(a)}(p, k; E) \\ & - \frac{8}{3\pi} \int_0^\Lambda dq K^{(a)}(p, q; E) \frac{\gamma + \sqrt{\gamma^2 + \frac{3}{4}(q^2 - k^2)}}{1 - \frac{1}{2}\rho_d \left(\gamma + \sqrt{\gamma^2 + \frac{3}{4}(q^2 - k^2)} \right)} \frac{q^2 T(q, k)}{q^2 - k^2 - i\epsilon}. \end{aligned} \quad (9)$$

3.2. Integral equation for scattering length

The scattering length, a_4 , of the quartet nd scattering is obtained by the on-shell scattering amplitude with zero momentum as

$$T(0, 0) = -\frac{3\pi}{m_N} a_4. \quad (10)$$

Now we introduce a half off-shell amplitude (or a half off-shell scattering length) as

$$T(p, 0) = -\frac{3\pi}{m_N} a(p, 0), \quad (11)$$

where $a(0, 0) = a_4$. Thus we have an integral equation in terms of $a(p, 0)$ as

$$\begin{aligned} a(p, 0; \Lambda) = & \frac{8Z_d}{3\rho_d} K^{(a)}(p, 0; -B_2) \\ & - \frac{8}{3\pi} \int_0^\Lambda dq K^{(a)}(p, q; -B_2) \frac{\gamma + \sqrt{\gamma^2 + \frac{3}{4}q^2}}{1 - \frac{1}{2}\rho_d \left(\gamma + \sqrt{\gamma^2 + \frac{3}{4}q^2} \right)} a(q, 0; \Lambda), \end{aligned} \quad (12)$$

where we have included the Λ dependence in the scattering length $a(p, 0; \Lambda)$ in the above expression though $a(p, 0; \Lambda)$ is insensitive to Λ when $\Lambda \simeq m_\pi$ or larger. B_2 is the deuteron binding energy, $B_2 = \gamma^2/m_N$.

Now we choose $\Lambda = 197 \text{ MeV}$, and solve Eq. (12) numerically. We have two input parameters, γ and ρ_d , whose values are known as $\gamma = 45.7 \text{ MeV}$ and $\rho_d = 1.76 \text{ fm}$, and thus have

$$a_4^{th} \equiv a(0, 0; \Lambda = 197 \text{ MeV}) = 6.34 \text{ fm}. \quad (13)$$

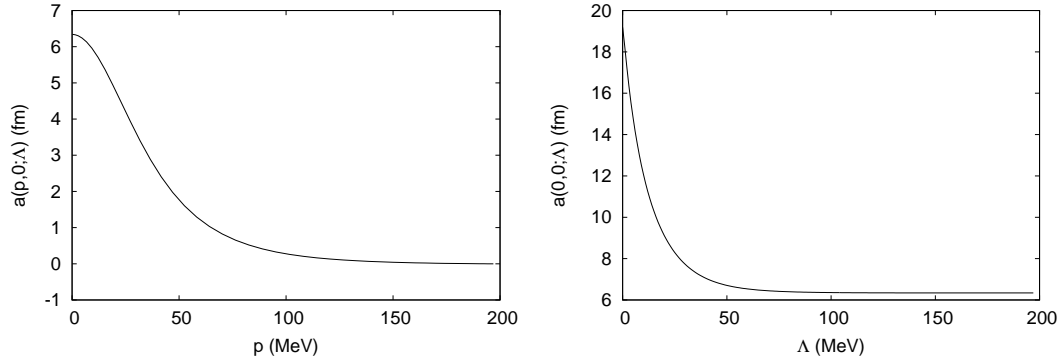


Figure 3: Left panel: Half off-shell $a(p, 0; \Lambda)$ (fm) with $\Lambda = 197$ MeV as a function of off-shell momentum p (MeV). Right panel: Cutoff dependence of scattering length $a(0, 0; \Lambda)$ (fm) without the three-body interaction.

We reproduce results obtained by Bedaque, Hammer, and van Kolck [13, 14], $a_4 = 6.33 \pm 0.10$ fm, and Griesshammer [19], 6.354 ± 0.020 fm, up to NNLO in pionless EFT. It agrees well with those obtained by Bedaque and Griesshammer [20], 6.8 ± 0.7 fm, up to NLO in EFT with perturbative pions, H. Witala *et al.* [21], $6.321 \dots 6.347$ fm, from various combinations of modern NN potentials and three-body forces, and the experimental datum [22],

$$a_4^{exp.} = 6.35 \pm 0.02 \text{ fm}. \quad (14)$$

In the left panel of Fig. 3, we numerically calculate and plot the half off-shell scattering length, $a(p, 0; \Lambda)$, at $\Lambda = 197$ MeV as a function of the off-shell momentum p . We find that the off-diagonal part of the scattering length quickly decreases and becomes almost negligible at $p > m_\pi$. This fact may indicate the insensitivity of the amplitude for the quartet channel to the value of Λ when $\Lambda = m_\pi$ or larger, $\Lambda \geq m_\pi$.

We now reduce the value of Λ . In the limit where Λ is sent to zero, we have the on-shell scattering length $a(0, 0; \Lambda)$ as

$$\lim_{\Lambda \rightarrow 0} a(0, 0; \Lambda) = \frac{8}{3} \frac{1}{\gamma(1 - \gamma\rho_d)} \simeq 19.19 \text{ fm}. \quad (15)$$

In the right panel of Fig. 3, we plot the cutoff Λ dependence of the scattering length $a(0, 0; \Lambda)$. One can see that there is almost no Λ dependence at $\Lambda > 100$ MeV, whereas the significant cutoff dependence appears in the calculated scattering length $a(0, 0; \Lambda)$ when the value of the cutoff is reduced less than γ ($= 45.7$ MeV).

3.3 Integral equation with three-body contact interaction

At the small cutoff region, especially being smaller than γ , the scattering length $a(0, 0; \Lambda)$ has the significant cutoff dependence. To make the result cutoff independent,

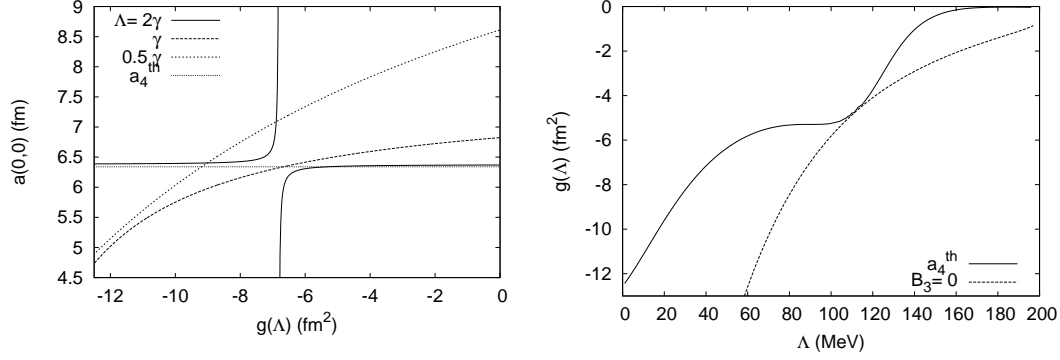


Figure 4: Left panel: $a(0,0)$ (fm) in the cases of three fixed cutoff values $\Lambda = 2\gamma$, γ , and 0.5γ as functions of $g(\Lambda)$ (fm²). A dashed horizontal line for a_4^{th} is also included. Right panel: $g(\Lambda)$ (fm²) which reproduce a_4^{th} (curve) and three-body bound state with $B_3 = 0$ (dashed curve) as functions of Λ (MeV).

as discussed before, we introduce the three-body contact interaction $g(\Lambda)$ which is supposed to take account of physics integrated out due to the small cutoff, and thus have the integral equation as

$$a(p, 0) = \frac{8Z_d}{3\rho_d} \left[\frac{1}{\gamma^2 + p^2} + g(\Lambda) \right] - \frac{8}{3\pi} \int_0^\Lambda dq \left[K^{(a)}(p, q; -B_2) + g(\Lambda) \right] \frac{\gamma + \sqrt{\gamma^2 + \frac{3}{4}q^2}}{1 - \frac{1}{2}\rho_d \left(\gamma + \sqrt{\gamma^2 + \frac{3}{4}q^2} \right)} a(q, 0), \quad (16)$$

where we have removed the Λ dependence from the half off-shell scattering length $a(p, 0)$ because the Λ dependence from the integral should be cancelled by $g(\Lambda)$.

In the limit that the cutoff Λ becomes large, at $\Lambda = 197$ MeV, the coupling constant $g(\Lambda)$ should vanish, $g(\Lambda) \rightarrow 0$. On the other hand, in the limit that the cutoff Λ vanishes, one has

$$a_4^{th} = \frac{8}{3} \frac{Z_d}{\rho_d} \left(\frac{1}{\gamma^2} + g(0) \right), \quad (17)$$

where $g(0) = -12.56 \cdots \text{fm}^2$ to reproduce the value of $a(0, 0; \Lambda = 197 \text{ MeV})$ in Eq. (13). Thus we consider a range of $g(\Lambda)$, $-12.56 \leq g(\Lambda) \leq 0$ (fm²), in this work.

In the left panel of Fig. 4, we numerically calculate from Eq. (16) and plot curves of $a(0, 0)$ with three fixed values of Λ , $\Lambda = 2\gamma$, γ , and 0.5γ as functions of $g(\Lambda)$. A dashed horizontal line for a_4^{th} is also included in the figure. One can notice that the curve for $\Lambda = 2\gamma$ is insensitive to $g(\Lambda)$ except for a singular point which corresponds to a three-body bound state with zero binding energy, $B_3 = 0$, whereas the curves with $\Lambda = \gamma$ and 0.5γ vary widely and smoothly.

In the right panel of Fig. 4, we numerically calculate from Eq. (16) and plot curves of $g(\Lambda)$ which reproduce a_4^{th} (curve) and the three-body bound state with $B_3 = 0$ (dashed curve) as functions of Λ . One can see that the three-body bound state with $B_3 = 0$ appears when the strength of $g(\Lambda)$ becomes stronger than that of $g(\Lambda)$ which reproduces a_4^{th} and Λ is larger than about 60 MeV in the figure. We find that the curve of $g(\Lambda)$ for the three-body bound state with $B_3 = 0$ varies smoothly, whereas that of $g(\Lambda)$ which reproduces a_4^{th} has a plateau like shape at the middle of the range of Λ , $\Lambda \simeq 60 \sim 100$ MeV. We use the curve of $g(\Lambda)$ which reproduces a_4^{th} when studying the perturbation expansions, the Born series, in the following.

4. Perturbative expansions: the Born series

Now we expand the scattering length and the amplitude in terms of the Born series around the weak coupling limit and the unitary limit, as discussed in the introduction.

4.1 The Born series for the scattering length

Firstly, we study the expansion in terms of the Born series for the scattering length a_4 . Thus the scattering length $a(0, 0)$ in Eq. (16) is expanded as

$$a_4 = a(0, 0) = \frac{8Z_d}{3\rho_d} [b_0 + b_1 + b_2 + \cdots], \quad (18)$$

where

$$b_0 = \frac{1}{\gamma^2} + g(\Lambda), \quad (19)$$

$$b_1 = \int_0^\Lambda dq \left[\frac{1}{\gamma^2 + q^2} + g(\Lambda) \right]^2 F(q), \quad (20)$$

$$b_2 = \int_0^\Lambda dq \left[\frac{1}{\gamma^2 + q^2} + g(\Lambda) \right] F(q) \\ \times \int_0^\Lambda dq' [K^{(a)}(q, q', -B_2) + g(\Lambda)] F(q') \left[\frac{1}{\gamma^2 + q'^2} + g(\Lambda) \right], \quad (21)$$

with

$$F(q) = -\frac{8}{3\pi} \frac{\gamma + \sqrt{\gamma^2 + \frac{3}{4}q^2}}{1 - \frac{1}{2}\rho_d \left(\gamma + \sqrt{\gamma^2 + \frac{3}{4}q^2} \right)}. \quad (22)$$

This expansion corresponds to that around the weak coupling limit. On the other hand, we consider another expansion around the inverse of the scattering length as

$$\frac{1}{a_4} = \frac{3\rho_d}{8Z_d} \left[\frac{1}{b_0} - \frac{b_1}{b_0^2} - \frac{b_2}{b_0^2} + \frac{b_1^2}{b_0^3} + \cdots \right]. \quad (23)$$

This one corresponds to that around the unitary limit.

In the left and right panel of Fig. 5, we numerically calculate and plot curves of the scattering length a_4 expanded around the weak coupling limit and the unitary limit,

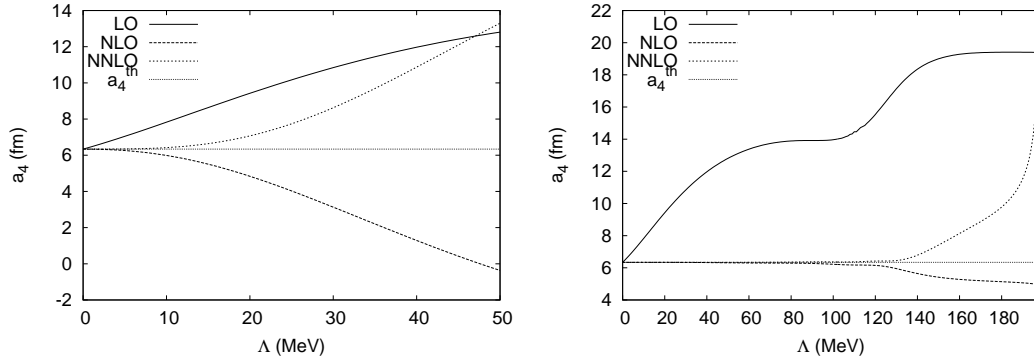


Figure 5: Left panel: Scattering length a_4 expanded around weak coupling limit as functions of Λ (MeV). Curves labeled by “LO” are results at leading order, “NLO” up to next-to-leading order, and “NNLO” up to next-to-next-to leading order in the both panels. A horizontal line of a_4^{th} is also included. Right panel: Scattering length a_4 expanded around unitary limit as functions of Λ (MeV).

respectively, as functions of Λ . Curves labeled by “LO” are results which include only the leading order term, b_0 , those by “NLO” are results up to next-to-leading order (NLO) which include first two terms in the brackets in Eq. (18) and (23), and those by “NNLO” are results up to next-to-next-to leading order (NNLO) which include all terms in the brackets.

In the left panel of Fig. 5, one can see that a region of the cutoff Λ , where the curves of a_4 obtained from the terms up to NLO and NNLO expanded around the weak coupling limit agree with a_4^{th} , is quite small, up to about 10 MeV. It would be a natural consequence of the perturbation around the weak coupling limit because the perturbation would converge when the scale Λ becomes smaller than a typical scale of the process. In the present case, it may be a_4^{th} ($1/a_4^{th} \simeq 31.4$ MeV), and thus it converges when the cutoff Λ becomes much smaller than $1/a_4^{th}$. In the right panel of Fig. 5, on the other hand, we find that the region where a_4^{th} is reproduced is broaden, up to about 100 MeV due to the two or three terms (up to NLO and NNLO, respectively) of the Born series expanded around the unitary limit.

4.2 The Born series for the scattering amplitude

Before expanding the integral equation for the scattering amplitude in terms of the Born series, we check how the three-body interaction $g(\Lambda)$ can reproduce the phase shift δ_0 at the small cutoff values. In Fig. 6, we numerically calculate and plot the phase shift δ_0 below deuteron breakup threshold as functions of the on-shell momentum k by solving the integral equation at small cutoff values, $\Lambda = 197, 140, 100$, and 60 MeV. The deuteron breakup momentum is $k_{br} \simeq 52.7$ MeV. We also include results from a modern potential model (Av18) [23] in the figure. One can see that the results are fairly independent on the cutoff values and considerably well agree to those obtained from the accurate Av18

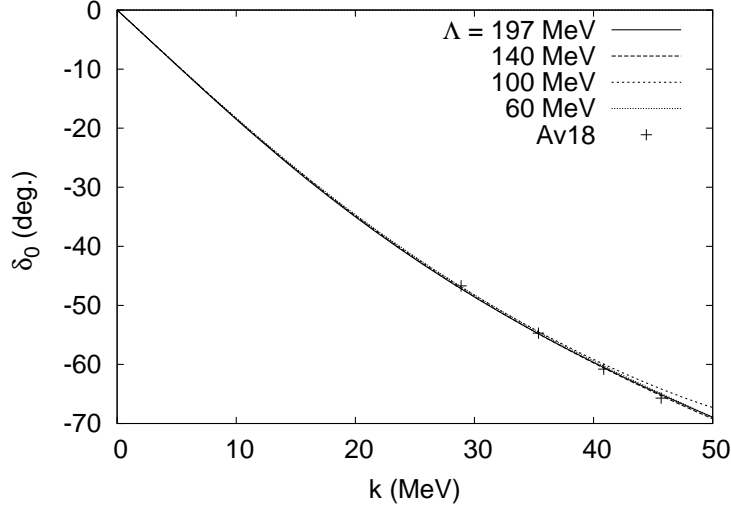


Figure 6: Phase shift δ_0 (deg.) of the S -wave nd scattering for spin quartet channel below deuteron breakup threshold as functions of k (MeV). Curves are obtained by solving the integral equation at cutoff values, $\Lambda = 197, 140, 100$, and 60 MeV with the three-body interaction $g(\Lambda)$ renormalized by a_4^{th} . Plus signs "+" denote results from a modern potential model (Av18) [23]. The deuteron breakup momentum is $k_{br} \simeq 52.7$ MeV.

potential model.

We now expand the on-shell T-matrix in terms of the Born series up to NNLO around the weak coupling limit and the unitary limit. Thus the on-shell T-matrix is obtained as

$$T(k, k) = -\frac{8\pi Z_d}{m_N \rho_d} [B_0 + B_1 + B_2 + \dots], \quad (24)$$

with

$$B_0 = V(k, k) = K^{(a)}(k, k; E) + g(\Lambda), \quad (25)$$

$$B_1 = \int_0^\Lambda dq V(k, q) G(q, k) V(q, k), \quad (26)$$

$$B_2 = \int_0^\Lambda dq V(k, q) G(q, k) \int_0^\Lambda dq' V(q, q') G(q', k) V(q', k), \quad (27)$$

where

$$G(q, k) = -\frac{8}{3\pi} \frac{\gamma + \sqrt{\gamma^2 + \frac{3}{4}(q^2 - k^2)}}{1 - \frac{1}{2}\rho_d [\gamma + \sqrt{\gamma^2 + \frac{3}{4}(q^2 - k^2)}]} \frac{q^2}{q^2 - k^2 - i\epsilon}. \quad (28)$$

This corresponds to the expansion around the weak coupling limit. We also have an expansion around the inverse of the T-matrix as

$$\frac{1}{T(k, k)} = -\frac{m_N \rho_d}{8\pi Z_d} \left[\frac{1}{B_0} - \frac{B_1}{B_0^2} - \frac{B_2}{B_0^3} + \frac{B_1^2}{B_0^3} + \dots \right]. \quad (29)$$

This expansion corresponds to that around the unitary limit.

To calculate the phase shift δ_0 from the Born series around the weak coupling limit in Eq. (24) we employ the relation

$$\delta_0 = \frac{1}{2i} \ln \left[1 + i \frac{2km_N}{3\pi} T(k, k) \right], \quad (30)$$

We note that when the amplitude expanded in the Born series is truncated, it breaks unitary condition and the phase shift becomes a complex number [24]. On the other hand, to calculate the phase shift from the expansion around the unitary limit in Eq. (29) we use the formula

$$k \cot \delta_0 = \frac{3\pi}{m_N} \text{Re} \frac{1}{T(k, k)}, \quad (31)$$

with

$$\text{Re} \frac{1}{T_{NLO}(k, k)} = -\frac{m_N \rho_d}{8\pi Z_d} \left[\frac{1}{B_0} - \frac{\text{Re} B_1}{B_0} \right], \quad (32)$$

$$\text{Re} \frac{1}{T_{NNLO}(k, k)} = -\frac{m_N \rho_d}{8\pi Z_d} \left[\frac{1}{B_0} - \frac{\text{Re} B_1}{B_0} - \frac{\text{Re} B_2}{B_0} + \frac{(\text{Re} B_1)^2 - (\text{Im} B_1)^2}{B_0^3} \right], \quad (33)$$

where B_0 has real part only. This expansion preserves the unitary condition, at least up to NNLO.

In Fig. 7, we numerically calculate and plot real and imaginary part of the phase shift δ_0 of S -wave quartet nd scattering obtained from the truncated Born series expanded around the weak coupling limit in Eq. (24). Curves up to NLO are obtained by including first two terms, B_0 and B_1 , and those up to NNLO by including all three terms, B_0 , B_1 , and B_2 , in Eq. (24). We have fixed Λ at $\Lambda = 13$ MeV because we found in the left panel of Fig. 5 that the scattering length a_4^{th} are fairly well reproduced when the cutoff value is about up to 10 MeV. A line labeled by “Full”³ obtained from the calculation without the truncation at $\Lambda = 197$ MeV is also included in the figure. One can see that the real parts of δ_0 agree with the full result up to about 6 MeV for NLO and 9 MeV for NNLO. In addition, the imaginary parts sharply decrease around the edge of the cutoff value $\Lambda = 13$ MeV, and it indicates that the unitary condition is broken. The broken unitary condition up to NLO is also partly cured by including the higher order term, B_2 , at NNLO.

In Figs. 8 and 9, we numerically calculate and plot curves of the phase shift δ_0 obtained from the Born series up to NLO and NNLO in Eqs. (32) and (33), respectively, expanded around the unitary limit where the cutoff values are chosen $\Lambda = 140, 100$, and 60 MeV. In addition, plus signs “+” labeled by “Full” in the figures denote the results from the calculation without the truncation at $\Lambda = 197$ MeV. We find in Fig. 8 that the curves considerably well converge to that of the full calculation as the cutoff value decreases,

³This line is the same as that at $\Lambda = 197$ MeV in Fig. 6.

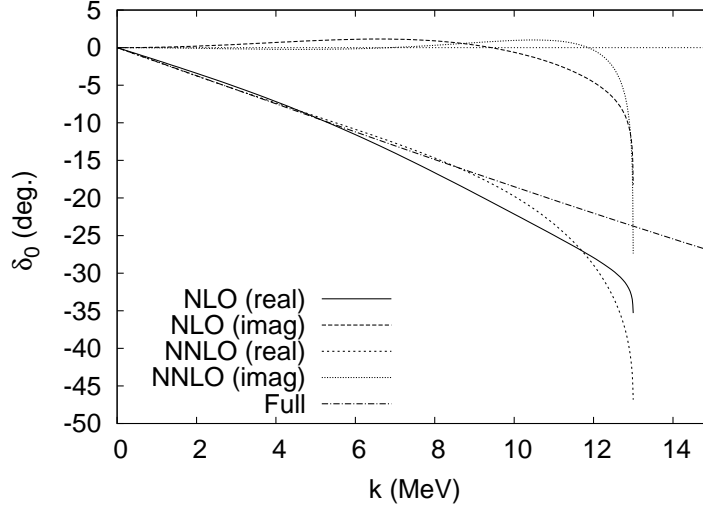


Figure 7: Real and imaginary part of phase shift δ_0 (deg.) of S -wave quartet nd scattering, as functions of k (MeV), obtained from the truncated Born series up to NLO and NNLO expanded around the weak coupling limit where the cutoff is fixed at $\Lambda = 13$ MeV. A dashed line labeled by “Full” obtained from the calculation without the truncation and $\Lambda = 197$ MeV is also included.

and the truncated Born series expanded around the unitary limit up to NLO fairly well reproduces the result of the full calculation at $\Lambda = 60$ MeV. In Fig. 9, we can see that the convergence to the full result, as reducing the cutoff value, becomes faster due to the inclusion of the higher order terms.

5. Discussion and conclusions

In this work, we studied the perturbative expansions, as the Born series, of the integral equation around the weak coupling limit and the unitary limit for the S -wave nd scattering for the spin quartet channel below the deuteron breakup threshold in pionless EFT at the small cutoff values. The three-body contact interaction is introduced when the integral equation is solved by using the small cutoff values. After the strength of the three-body interaction is renormalized by using the scattering length a_4 , we expand the integral equation for the scattering length and the amplitude, as the Born series, up to NNLO around the weak coupling limit and the unitary limit. We find that the scattering length and the phase shift are considerably well reproduced by a few terms of the Born series as we reduce the cutoff values to about 10 MeV for the expansion around the weak coupling limit and to about 60 MeV for that around the unitary limit.

In the present particular process, the S -wave quartet nd scattering studying, one may regard that there is no advantage by sending Λ to small values because the physical observables, the scattering length a_4 and the phase shift δ_0 , are well described (without fitting any unknown parameters) by the two effective range parameters in the deuteron

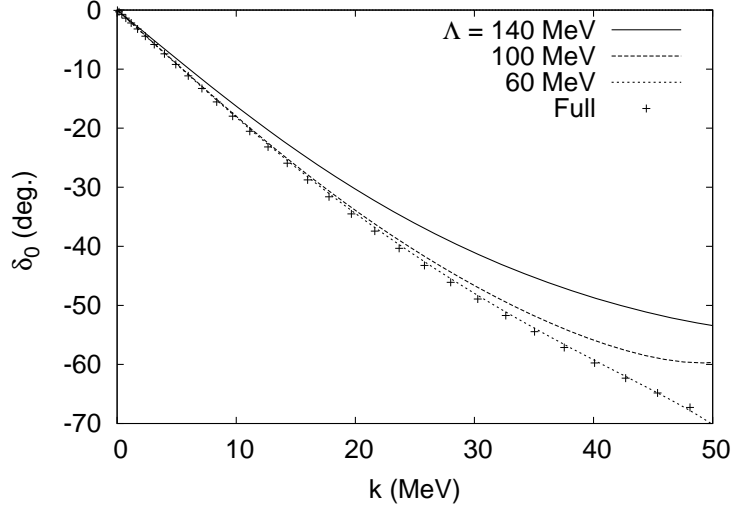


Figure 8: Phase shift δ_0 (deg.) of S -wave quartet nd scattering, as functions of k (MeV), obtained from the Born series up to NLO expanded around the unitary limit with $\Lambda = 140, 100$, and 60 MeV. Plus signs “+” labeled by “Full” are obtained from the calculation without the truncation at $\Lambda = 197$ MeV.

channel at the usual large scale of the pionless theory, $\Lambda \simeq m_\pi$. However, it could be a useful limit when one studies a relation between pionless EFT and a Halo/Cluster EFT whose large scale smaller than m_π , such as a deuteron cluster theory for the nd scattering below the deuteron breakup momentum.

In addition, it may be an interesting observation that the physical observables can be well described by a few terms of the Born series expanded around the unitary limit at the small cutoff values. If this property were common in some class of the interactions and/or in the unitary limit, it could provide us a useful method to make a non-perturbative interaction perturbative and/or to be used in studies of the exotic nuclei near the drip line. Indeed, one may regard that what we found in this work can be a coincidence, a unique property only in the S -wave quartet nd scattering or in the case of “non-singular” interactions. Therefore, it may be worth studying whether the same property which we found in this work can be observed or not in the case of the singular interactions, such as the S -wave spin doublet nd scattering or the NN scattering involving the tensor force in the 3S_1 - 3D_1 channel.

Acknowledgements

The author would like to thank K. Kubodera for reading the manuscript. This work is supported by the Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (2010-0023661) and (2012R1A1A2009430).

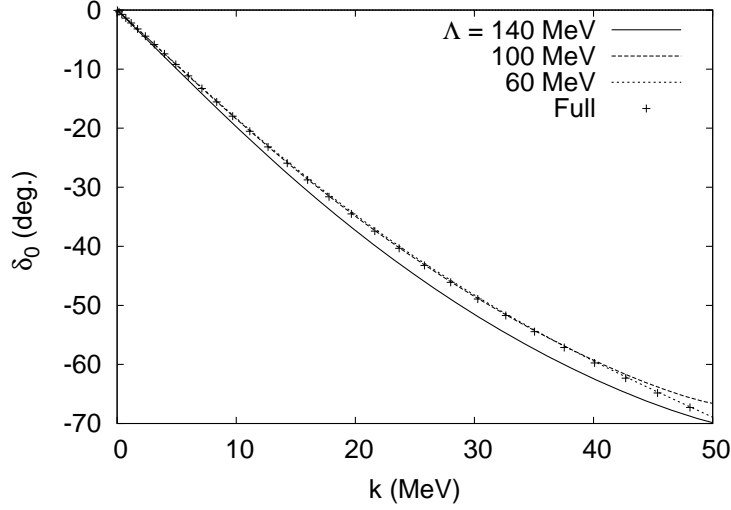


Figure 9: Phase shift δ_0 (deg.) obtained from the Born series up to NNLO expanded around the unitary limit with $\Lambda = 140, 100$, and 60 MeV. See the caption of Fig. 8 as well.

References

- [1] S. Weinberg, Phys. Lett. B 251, 288 (1990), Nucl. Phys. B 363, 1 (1991).
- [2] P.F. Bedaque and U. van Kolck, Annu. Rev. Nucl. Part. Sci. 52, 339 (2002).
- [3] E. Braaten and H.-W. Hammer, Phys. Rep. 428, 259 (2006).
- [4] E. Epelbaum, H.-W. Hammer, U.-G. Meissner, Rev. Mod. Phys. 81, 1773 (2009).
- [5] R. Machleidt and D.R. Entem, Phys. Rep. 503, 1 (2011).
- [6] B. Jonson, Phys. Rep. 389, 1 (2004).
- [7] M.V. Zhukov *et al.*, Phys. Rep. 231, 151 (1993).
- [8] I. Tanihata, J. Phys. G: Nucl. Part. Phys. 22, 157 (1996).
- [9] J.-W. Chen, G. Rupak, M.J. Savage, Nucl. Phys. A 653, 386 (1999).
- [10] S. Ando and C. H. Hyun, Phys. Rev. C 72, 014008 (2005).
- [11] P. F. Bedaque, H.-W. Hammer, U. van Kolck, Phys. Rev. Lett. 82, 463 (1999).
- [12] P. F. Bedaque, H.-W. Hammer, U. van Kolck, Nucl. Phys. A 676, 357 (2000).
- [13] P. F. Bedaque and U. van Kolck, Phys. Lett. B 428 (1998) 221.

- [14] P. F. Bedaque, H.-W. Hammer, U. van Kolck, Phys. Rev. C 58 (1998) R641.
- [15] C.A. Bertulani, H.-W. Hammer, U. van Kolck, Nucl. Phys. A 712, 37 (2002).
- [16] S.-I. Ando, in progress.
- [17] M.C. Birse, J.A. McGovern, K.G. Richardson, Phys. Lett. B 464, 169 (1999).
- [18] S.-I. Ando, arXiv:1209.3070 [nucl-th].
- [19] H.W. Griesshammer, Nucl. Phys. A 744 (2004) 192.
- [20] P.F. Bedaque and H.W. Griesshammer, Nucl. Phys. A 671 (2000) 367.
- [21] H. Witala *et al.*, Phys. Rev. C 68 (2003) 034002.
- [22] W. Dilg, L. Koester, and W. Nistler, Phys. Lett. 36B (1971) 208.
- [23] A. Kievsky *et al.*, Nucl. Phys. A 607 (1996) 402.
- [24] S.-I. Ando and C.H. Hyun, Phys. Rev. C 86, 024002 (2012).